## ARTICLE VIII.

*On the Purturbations of Meteors approaching near the Earth. By Benjamin Pierce, A. A., Hollis Professor of Mathematics and Natural Philosophy in Harvard University; in a Letter to S. C. Walker, Esq. Read January 15, 1841.*

MY DEAR SIR,

MANY sources of almost unceasing occupation have prevented my giving Mr. Erman's* paper on Meteors that early attention which I intended. I shall now turn my attention to the point which you suggested of the earth's attraction. Almost the whole labour has fortunately been saved for me by Laplace, in the Méc. Cél., Vol. IV., Book IX., Chap. II., "On the Perturbations of a Comet, when it approaches very near to a Planet." He has there proved (8038, Bowd. Ed.,) that "we may, in the calculation of such a comet, suppose the planet to have a sphere of activity, in which the *relative* motion of the comet is affected only by the planet's attraction; and that beyond this point the *absolute* motion of the comet about the sun is performed in exactly the same manner as if the sun alone acted upon it." The radius of this sphere is by (8035)

$$r.\sqrt[5]{(\tfrac{1}{2}\,m^2)};$$

in which

$$r = \text{the radius vector of the planet,}$$
$$m = \text{its mass divided by the sun's mass;}$$

so that, in the present case, this radius, which we denote by $r_0$, is

$$r_0 = 0{\cdot}0053,$$

in which the unit is the same as Erman's.

Now the relative orbit of the meteoric ring is directed nearly to the centre

* Schumacher's Astr. Nachr., No. 385.

of the earth, and I shall regard it as exactly directed towards this centre; in which case, the only effect of the earth's attraction is to increase the relative velocity without changing the relative direction. The only change which is, therefore, required in Erman's paper is to increase the relative velocity $v'$ by this increase of velocity. The increased velocity is determined by the formula

$$v_0'^2 = v'^2 + \frac{2g}{R} - \frac{2g}{r_0}$$

in which

$R =$ the earth's radius,
$g =$ the attractive force of the earth at the unit of distance,
$v_0' =$ the increased velocity.

Hence

$$\frac{2g}{R} - \frac{2g}{r_0} = 0{\cdot}13932, \qquad v_0'^2 = v'^2 + 0{\cdot}13932$$

and the five values of $v_0'$, corresponding to the five values of $v'$, calculated by Erman, are

0·91118, 1·41818, 1·69177, 1·93830, 2·17363.

Nothing farther seems needed upon this point, and I therefore leave it to notice an omission by Professor Erman.

He has neglected the negative sign of the radical in the equation

$$v' = - e \cos. u \pm \sqrt{(v^2 - e^2 \sin.^2 u)},$$

and this sign may be used as long as the resulting value of $v'$ is not negative. Thus, for the value of $v$

$$= 0{\cdot}77382$$

we should find

$$\begin{aligned} v' &= 0{\cdot}29426 \\ \text{velocity in perihelion} &= 2{\cdot}235 \\ \text{perihelion distance} &= 0{\cdot}4186 \\ T &= 0{\cdot}60973 \\ \omega &= 12^\circ\ 3' \end{aligned}$$

which shows that Erman's conclusions, regarding the relative velocity and the inclination of the orbit, are unsound.

In reviewing some of his numerical results, I differ a little from him, but the difference is of no practical importance.

Another most important point for consideration is the difference of direction

of the different meteors. Now this difference of direction amounts to more than 10° on the average (according to Erman) from the mean direction, and cannot, for the maximum, be less than 25°. This difference of direction may arise

1. From the difference in their elliptic orbits about the sun.
2. From their mutual action.
3. From the earth's attraction.

1. Supposing, with Erman, the breadth of the ring to be 2°, the difference arising from the first cause cannot be more than 1° from the mean direction.

2. The deviations arising from their mutual attractions must be trifling; they cannot, for instance, be supposed greater than they would be if all the meteors but the disturbed one, which we may be considering, were combined into one planet, about which this disturbed one moved as a satellite. Now if we consider that the variation in the moon's *absolute* direction from the earth's is only about 2°, we shall have no hesitation in neglecting this second cause of disturbance.

The difference in direction arising from the first two causes is absolute, and may be somewhat magnified when converted into relative direction, but not much, unless the relative velocity $v'$ is very small. There is also a difference of absolute velocity, which will produce a difference of relative direction of about the same order of magnitude with that arising from the difference of absolute direction.

3. The observed difference of direction must then be chiefly referred to the principal disturbing cause, the earth; and the following method of calculation is sufficiently accurate for the present case. Let

$v'$ = the relative velocity of the meteors at the moment of entering the sphere of the earth's influence, which sphere we may, for this calculation, suppose to be infinite,

$\phi$ = the angular deflection of the meteor's relative motion,

then no other meteor will be so much deflected as the one which just grazes the surface of the earth; and for this meteor we have

$$\operatorname{cosec.} \phi = 1 + \frac{v'^2 R}{m}$$

$$v'^2 = \frac{m}{R} (\operatorname{cosec.} \phi - 1)$$

whence, for a deviation of 15°, $v'$ must be less than one-third of the earth's velocity; that is, far less than either of Erman's supposed values of $v'$; and for the least value of $v'$, which Erman has given, namely,

$$v' = 0{\cdot}83122,$$

we have

$$\phi = 2^\circ\ 38',$$

which appears to be entirely contrary to a large majority of the observations.

Now, for

$$v' = 0{\cdot}33333 = \tfrac{1}{3}$$

we find

$$w = 14^\circ\ 10', \quad v = {\cdot}75 = \tfrac{3}{4}$$

so that the plane of the meteors cannot differ much from that of the ecliptic, and their relative velocity cannot exceed one-third of the earth's velocity. The other elements of the orbit are of less interest, and I shall not stop to calculate them. A ring so nearly in the plane of the earth's orbit must be subject to very great perturbations, and if there is one, I think that no observations which we can make will enable us to calculate its motions with any degree of accuracy.

Believe me, my dear sir,

most sincerely yours,

B. PIERCE.

CAMBRIDGE, *December* 24, 1840.